

SAYFA-1

KARAKÖK

$$1) x^2 = a^2 \Rightarrow x_{1,2} = \pm \sqrt{a}$$

$$2) a < 0 \Rightarrow y = \sqrt{a^2} = |a| = -a > 0$$

NOT= Karaköklü ifadenin içi hiç bir zaman negatif olmaz.

$$3) \sqrt[n]{x} = n. \text{ kuvvetten kök } x \text{ (x sayısının n. kuvvetten kökü)}$$

$$\begin{aligned} n=2 &\Rightarrow \text{karekök} \\ n=3 &\Rightarrow \text{küpkök} \end{aligned}$$

$$4) \sqrt[n]{x^m} = x^{\frac{m}{n}} = x^{\frac{m \cdot p}{n \cdot p}} \quad x, m, n, p \in \mathbb{R}$$

$$\left. \begin{aligned} 5) a, b \in \mathbb{R} \\ a \geq 0, b \geq 0 \\ n=2k \text{ veya } n=2k+1 \\ k \in \mathbb{N} \end{aligned} \right\} \Rightarrow \sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$6) x^n = y = \underbrace{x \cdot x \cdot x \dots x}_{n \text{ tane}} \Rightarrow x = \sqrt[n]{y}$$

$$\left. \begin{aligned} 7) a, b \in \mathbb{R} \\ a \geq 0, b > 0 \\ n=2k, k \in \mathbb{N} \end{aligned} \right\} \Rightarrow \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$8) \frac{1}{\sqrt[n]{a}} = \frac{\sqrt[n]{a^{n-1}}}{a}, \quad a < b \Rightarrow \sqrt[n]{a} < \sqrt[n]{b}, \quad (\sqrt[n]{a})^n = \sqrt[n]{a^n}$$

$$\left. \begin{aligned} 9) a, b \in \mathbb{R} \\ m \in \mathbb{R} \end{aligned} \right\} \Rightarrow a\sqrt[m]{m} + b\sqrt[m]{m} = (a+b)\sqrt[m]{m}$$

$$10) x^m \cdot x^n = x^{m+n}, \quad \frac{x^m}{x^n} = x^{m-n} = \frac{1}{x^{n-m}}$$

$$11) \sqrt[n]{a} \cdot \sqrt[n]{a} = \sqrt[n]{a^m} \cdot \sqrt[n]{a^n} = \sqrt[n]{a^{m+n}}$$

$$12) \sqrt[n]{a^k} \cdot \sqrt[n]{b^l} = \sqrt[n]{a^{km} \cdot b^{ln}}$$

$$13) \sqrt[n]{a^k} \cdot \sqrt[n]{a^l} = \sqrt[n]{a^{mk+ln}}$$

$$14) a^m \cdot \sqrt[n]{b} = \sqrt[n]{b \cdot a^{m \cdot n}}$$

$$15) \sqrt[n]{a^{m+n}} = a \sqrt[n]{a^m}, \quad \sqrt[n]{a^{m+pn}} = a^p \sqrt[n]{a^m}$$

$$16) \sqrt[n]{\sqrt[n]{a}} = \sqrt[n \cdot n]{a}$$

$$17) \sqrt[n]{a \sqrt[n]{a \sqrt[n]{a}}} = \sqrt[n]{a^{2^n - 1}} \quad n=3 \text{ burdaki değeridir.}$$

n sonlu bir sayı ve iç içe n tane karakök varsa

$$18) y = \sqrt[n]{a \sqrt[n]{a} \sqrt[n]{a} \dots} = \sqrt[n-1]{a}$$

$$y^n = a \sqrt[n]{a \sqrt[n]{a} \dots}$$

$$y^n = a \cdot y \Rightarrow y^{n-1} = a$$

$$y = \sqrt[n-1]{a}$$

$$19) \left. \begin{aligned} y &= \sqrt{a + \sqrt{a + \dots}} \\ y^2 &= a + \sqrt{a + \dots} \end{aligned} \right\} \Rightarrow y^2 = a + y, \quad y^2 - y - a = 0 \text{ dan } y \text{ çözülür.}$$

$$20) m > n \Rightarrow \frac{1}{\sqrt[m]{x^n}} = \frac{\sqrt[m]{x^{m-n}}}{x}$$

$$21) \sqrt{a \mp \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \mp \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$$

$$22) \sqrt[n]{x} = \frac{1}{\sqrt[n]{x^{n-1}}}$$

$$23) (a \cdot \sqrt[m]{b})^n = a^n \cdot \sqrt[m]{b^n}$$

$$24) y = \sqrt{a \mp 2\sqrt{b}} \quad \left. \begin{aligned} b &= c \cdot d \\ a &= c + d \\ c > d, \frac{a^2}{4} > b \end{aligned} \right\} \Rightarrow y = \sqrt{c} \mp \sqrt{d}$$

$$25) y = \sqrt[n]{a \cdot \sqrt[n]{a} \sqrt[n]{a} \dots} = \sqrt[n+1]{a}$$

$$y^n = \frac{a}{y} \Rightarrow y^{n+1} = a \Rightarrow y = \sqrt[n+1]{a}$$

$$26) y = \sqrt[n]{a \mp \sqrt[n]{c} \sqrt[n]{b}} = d \mp \sqrt[n]{b} > 0 \quad a = d^2 + b, \quad d > \sqrt[n]{b}$$

$$= \sqrt[n]{b} \mp d > 0 \quad a = d^2 + b, \quad \sqrt[n]{b} > d$$

$$27) y = \sqrt[m]{a \sqrt[m]{a} \sqrt[m]{a} \dots} = \sqrt[m]{a^{\frac{m^n - 1}{m - 1}}}$$

$n = \text{ic, içe kök sayısı}$ (burda 3'e eşittir.)

$$28) y = \sqrt{a + \sqrt{a + \sqrt{a + \dots}}} = \frac{-1 \pm \sqrt{1+4a}}{2} \quad \begin{array}{l} \text{- de} \\ a \geq 1 \end{array} \quad \text{olmak şartıyla}$$

$$29) y = \sqrt{n \cdot (n+1) + \sqrt{n \cdot (n+1) + \dots}} = \frac{- \text{de } n}{da \quad n+1 \text{ 'e eşit}} \quad \begin{array}{l} + da \\ a \geq 0 \end{array}$$

$$30) y = \sqrt[n]{a \sqrt[n]{a} \sqrt[n]{a} \sqrt[n]{a} \dots} = \sqrt[n]{a^{m+1}}$$

$$31) y = \sqrt[n]{a^m \sqrt[n]{b} \sqrt[n]{a^m} \sqrt[n]{b} \dots} = \sqrt[n]{a^m b}$$

KARAKÖKLE İLGİLİ BAZI KURALLARIN İSPATI

$$1) y = \sqrt{a + \sqrt{a + \sqrt{a + \dots}}} \quad a \geq 0 \Rightarrow y = \frac{1 + \sqrt{1+4a}}{2} \quad \text{dir}$$

İSPAT=

$$(y)^2 = \left(\sqrt{a + \underbrace{\sqrt{a + \sqrt{a + \dots}}}_y} \right)^2 \Rightarrow \text{Tanımlı ise } a \geq 0$$

$$y^2 = a + y \Rightarrow y^2 - y - a = 0 = a_1 y^2 + b_1 y + c_1 \Rightarrow \begin{array}{l} a_1 = 1 \\ b_1 = -1 \\ c_1 = -a \end{array}$$

$$\Delta = b_1^2 - 4a_1 c_1$$

$$\Delta = 1 + 4a \geq 0 \Rightarrow 4a \geq -1 \text{ şartı var}$$

$$y_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$y_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{1 + \sqrt{1+4a}}{2a} \geq 0 \quad y > 0$$

$$\boxed{y = y_1 = \frac{1 + \sqrt{1+4a}}{2} \text{ alınır}}$$

$$y_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{1 - \sqrt{1+4a}}{2}$$

$a \geq 0$ olmak şartıyla y nite tanımlı olabilmesi için $y_2 \geq 0$ olmak zorundadır.

$$y_2 \geq 0$$

$$1 - \sqrt{1+4a} \geq 0 \Rightarrow 1 + 4a \geq 0$$

$$(\sqrt{1+4a})^2 \leq (1)^2$$

SONUÇ=

$$1 + 4a \leq 1$$

$$4a \leq 0$$

$a \leq 0$, $a \geq 0$ ile geliştinden $y = y_1$ değeri dır.

$$\boxed{y = y_1 \neq y_2} \Rightarrow \text{BULUNUR}$$

SAYFA-4

KARAKÖKLE İLGİLİ BAZI KURALLARIN İSPATI

$$2) y = \sqrt{a - \sqrt{a - \sqrt{a - \dots}}} = \frac{-1 + \sqrt{1+4a}}{2} \Rightarrow \text{Tanımlıysa } a \geq 1 \text{ dir}$$

İSPAT=

$$(y)^2 = \left(\sqrt{a - \underbrace{\sqrt{a - \sqrt{a - \dots}}}_y} \right)^2 \quad \text{Tanımlıysa } a \geq 1$$

$$y^2 = a - y \Rightarrow y^2 + y - a = 0 = a_1 y^2 + b_1 y + c_1 \Rightarrow \begin{aligned} a_1 &= 1 \\ b_1 &= 1 \\ c_1 &= -a \end{aligned}$$
$$\Delta = b_1^2 - 4a_1 c_1$$
$$\Delta = 1 + 4a \geq 0 \Rightarrow 4a \geq -1 \text{ şartı var,}$$

$$y_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$y_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-1 + \sqrt{1+4a}}{2} \geq 0 \text{ olmak zorunda}$$

$$(\sqrt{1+4a}) \geq (1)$$

$$1+4a \geq 1$$

$$4a \geq 0 \Rightarrow a \geq 0$$

Not= $a \geq 1$ olduğunda $a > 0$ şartını da sağladığından

$$y = y_1 = \frac{-1 + \sqrt{1+4a}}{2} \text{ olur}$$

$$y_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-1 - \sqrt{1+4a}}{2} < 0 \text{ olacağından ve } y \text{ de } 1 \text{ köklü ifadenin içi negatif çıkamayacağından}$$

$$y = y_1, y \neq y_2 \text{ alınır}$$

SONUÇ=

$$y = \sqrt{a - \sqrt{a - \sqrt{a - \dots}}} = y_1 = \frac{-1 + \sqrt{1+4a}}{2} \text{ bulunur}$$

$$y \neq y_2 < 0$$

KARAKÖKLE İLGİLİ BAZI KURALLARIN İSPATI

$$3) y = \sqrt{a \mp \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \mp \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$$

İSPAT=

$$a \mp \sqrt{b} = a \mp \sqrt{a^2 - a^2 + b} = a \mp \sqrt{a^2 - (a^2 - b)}$$

$$a \mp \sqrt{b} = \frac{a}{2} + \frac{a}{2} + \frac{\sqrt{a^2 - b}}{2} - \frac{\sqrt{a^2 - b}}{2} \mp 2 \cdot \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \cdot \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$$

$$a \mp \sqrt{b} = \frac{a}{2} + \frac{\sqrt{a^2 - b}}{2} \mp 2 \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \cdot \sqrt{\frac{a - \sqrt{a^2 - b}}{2}} + \frac{a}{2} - \frac{\sqrt{a^2 - b}}{2}$$

$$a \mp \sqrt{b} = \frac{a + \sqrt{a^2 - b}}{2} \mp 2 \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \cdot \sqrt{\frac{a - \sqrt{a^2 - b}}{2}} + \frac{a - \sqrt{a^2 - b}}{2}$$

$$a \mp \sqrt{b} = \left(\sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \right)^2 \mp 2 \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \cdot \sqrt{\frac{a - \sqrt{a^2 - b}}{2}} + \left(\sqrt{\frac{a - \sqrt{a^2 - b}}{2}} \right)^2$$

$$(\sqrt{a \mp \sqrt{b}})^2 = \left(\sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \mp \sqrt{\frac{a - \sqrt{a^2 - b}}{2}} \right)^2 \Rightarrow \boxed{\begin{matrix} a > 0 \\ b > 0 \end{matrix} \quad \begin{matrix} a^2 = b^2 \Rightarrow a = b \\ \text{KURAL} \end{matrix}}$$

$$\sqrt{a \mp \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \mp \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$$

$a > 0, b > 0$
- de $a > \sqrt{b}$
+ da farketmez.

$$4) y = \sqrt{(a+b) \mp 2\sqrt{a \cdot b}} = \sqrt{a} \mp \sqrt{b} \quad \left. \begin{matrix} a > 0 \\ b > 0 \end{matrix} \right\} \begin{matrix} - \text{ de } a > b \\ + \text{ da } \text{farketmez} \end{matrix}$$

İSPAT=

$$y = \sqrt{a} \mp \sqrt{b}$$

$$y^2 = (\sqrt{a} \mp \sqrt{b})^2 = a \mp 2\sqrt{a} \cdot \sqrt{b} + b = (a+b) \mp 2\sqrt{a \cdot b}$$

$$\sqrt{y^2} = \sqrt{(a+b) \mp 2\sqrt{a \cdot b}} \geq 0$$

$$y = \sqrt{(a+b) \mp 2\sqrt{a \cdot b}} = \sqrt{a} \mp \sqrt{b}$$

SONUC=

$$y = \sqrt{(a+b) \mp 2\sqrt{ab}} = \sqrt{a} \mp \sqrt{b}$$

$\left. \begin{matrix} a > 0 \\ b > 0 \end{matrix} \right\} \Rightarrow \begin{matrix} - \text{ de } a > b \\ + \text{ da } \text{farketmez} \end{matrix}$

KARAKÖKLE İLGİLİ BAZI KURALLARIN İSPATI

$$5) y = \sqrt[n]{a^m \sqrt[n]{a^m \sqrt[n]{a^m \sqrt[n]{a^m \dots}}}} = \sqrt[mn-1]{a^{m+1}} \quad a \geq 0$$

İSPAT=

$$y = \sqrt[n]{a^m \sqrt[n]{a^m \sqrt[n]{a^m \sqrt[n]{a^m \dots}}}}$$

$$y^n = a^m \sqrt[n]{a^m \sqrt[n]{a^m \sqrt[n]{a^m \dots}}}$$

$$y^{mn} = a^m \cdot a^m \underbrace{\sqrt[n]{a^m \sqrt[n]{a^m \sqrt[n]{a^m \dots}}}}_y$$

$$\frac{y^{mn}}{y} = \frac{a^{m+1} \cdot y}{y}$$

$$y^{mn-1} = a^{m+1}$$

$$y = \sqrt[mn-1]{a^{m+1}} \quad a \geq 0 \quad m, n \in \mathbb{Z}^+$$

$$6) y = \sqrt[n]{a^m \sqrt[n]{b^m \sqrt[n]{a^m \sqrt[n]{b^m \sqrt[n]{a^m \sqrt[n]{b^m \dots}}}}}} = \sqrt[mn-1]{a^m \cdot b} \quad a, b \geq 0, \quad \begin{matrix} m, n \geq 1 \\ m, n \in \mathbb{Z}^+ \end{matrix}$$

İSPAT

$$y = \sqrt[n]{a^m \sqrt[n]{b^m \sqrt[n]{a^m \sqrt[n]{b^m \sqrt[n]{a^m \sqrt[n]{b^m \dots}}}}}}$$

$$y^n = a^m \sqrt[n]{b^m \sqrt[n]{a^m \sqrt[n]{b^m \sqrt[n]{a^m \sqrt[n]{b^m \dots}}}}}$$

$$y^{mn} = a^m b^m \underbrace{\sqrt[n]{a^m \sqrt[n]{b^m \sqrt[n]{a^m \sqrt[n]{b^m \dots}}}}_y$$

$$\frac{y^{mn}}{y} = \frac{a^m \cdot b^m \cdot y}{y}$$

$$y^{mn-1} = a^m \cdot b^m$$

$$y = \sqrt[mn-1]{a^m b^m} \quad a, b \geq 0 \quad m, n \in \mathbb{Z}^+ \quad mn \geq 1$$

SONUÇ=

$$y = \sqrt[n]{a^m \sqrt[n]{b^m \sqrt[n]{a^m \sqrt[n]{b^m \sqrt[n]{a^m \sqrt[n]{b^m \dots}}}}}} = \sqrt[mn-1]{a^m b^m} \quad a, b \geq 0 \quad \begin{matrix} mn \geq 1 \\ m, n \in \mathbb{Z}^+ \end{matrix}$$

KARAKÖKLE İLGİLİ BAZI KURALLARIN İSPATI

$$7) \quad y = \sqrt{n \cdot (n+1) + \underbrace{\sqrt{n \cdot (n+1) + \dots}}_y} = n+1 \quad \text{Tanımlı ise } n \geq 0$$

$$y^2 = n \cdot (n+1) + y$$

$$y^2 - y - n \cdot (n+1) = 0 = a_1 y^2 + b_1 y + c_1 \Rightarrow \begin{aligned} a_1 &= 1 \\ b_1 &= -1 \\ c_1 &= -n \cdot (n+1) \end{aligned}$$

$$\Delta = b_1^2 - 4a_1c_1 = 1 + 4 \cdot n \cdot (n+1) = 4n^2 + 4n + 1 = (2n+1)^2 \geq 0 \Rightarrow \text{Tanımlı}$$

$$\begin{array}{r} 2n \quad \quad 1 \\ 2n \quad \quad 1 \\ \hline 2n+2n=4n \end{array}$$

$$y_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{1 + \sqrt{(2n+1)^2}}{2} \geq 0 \text{ olduğundan } y \text{ Tanımlı}$$

$$y_1 = \frac{1 + (2n+1)}{2} = \frac{2 \cdot (n+1)}{2} = n+1, 0$$

$$y = y_1 = \sqrt{n \cdot (n+1) + \sqrt{n \cdot (n+1) + \dots}} = n+1 \text{ dir}$$

$$y_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{1 - \sqrt{(2n+1)^2}}{2} \geq 0 \Rightarrow y \text{ Tanımlıdır.}$$

$n \geq 0$ olduğunda y_2 değeri negatif çıkacağından $y \neq y_2$ dir.

$$y \neq y_2 = \frac{1 - \sqrt{(2n+1)^2}}{2} \text{ olarak alınır.}$$

$$8) \quad y = \sqrt{n \cdot (n+1) - \underbrace{\sqrt{n \cdot (n+1) - \dots}}_y} = n \quad \text{Tanımlı ise } n \geq 1 \text{ dir.}$$

$$y^2 = n \cdot (n+1) - y$$

$$y^2 + y - n \cdot (n+1) = 0 = a_1 y^2 + b_1 y + c_1 \Rightarrow \begin{aligned} a_1 &= 1 \\ b_1 &= 1 \\ c_1 &= -n \cdot (n+1) \end{aligned}$$

$$\Delta = b_1^2 - 4a_1c_1 = 1 + 4n \cdot (n+1) = 4n^2 + 4n + 1 = (2n+1)^2 > 1$$

$$y_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm \sqrt{(2n+1)^2}}{2}$$

$$y_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-1 + \sqrt{(2n+1)^2}}{2} > 0$$

$$y = y_1 = \frac{-1 + 2n + 1}{2} = n = \sqrt{n \cdot (n+1) - \sqrt{n \cdot (n+1) - \dots}} \text{ dir}$$

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KARAKÖKLE İLGİLİ BAZI KURALLARIN İSPATI

8) Devamı

$$y_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-1 - \sqrt{(2n+1)^2}}{2}$$

$n \geq 0$ ise

$2n+1 > 1$ olduğundan
 $y_2 < 0$ ve $y \neq y_2$ dir.

SONUÇ =

$$y = \sqrt{n(n+1)} - \sqrt{n(n+1)} - \dots = n \quad \text{Tanımlıysa} \quad n \geq 1 \text{ dir.}$$